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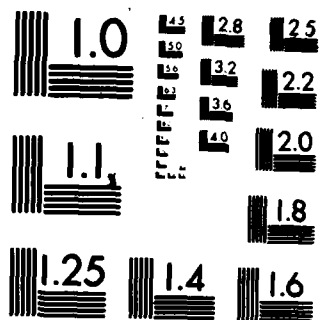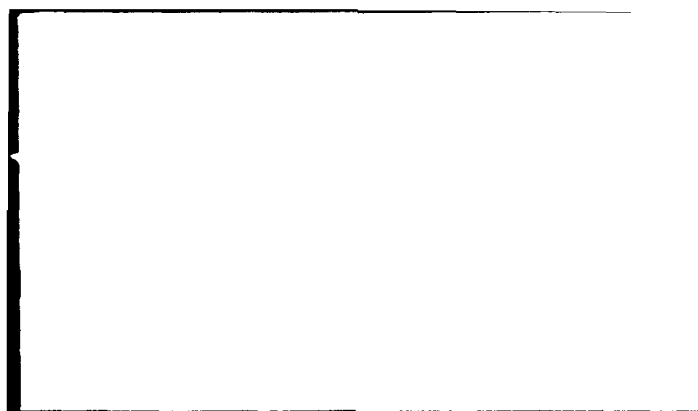   TR-113-11 N00014-79-C-0650
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Applied Research in Statistics - Mathematics - Operations Research

A NOTE ON THE ANALYSIS
OF RANDOM BALANCE DESIGNS

by

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1. INTRODUCTION AND BACKGROUND

In a two-level (± 1) Random Balance (RB) design for studying K

factors, each column of the NxK design matrix consists of N/2 +1's

and N/2 -1's where N (an even number) denotes the total number of

observations. In each design column the +1's and -1's are assigned

randomly, making all possible combinations of N/2 +1's and N/2 -1's

(there are $C_{N/2}^{N}$ in all) equally likely, with each column receiving

an independent randomization. The RB method is discussed at length

by Satterthwaite (1959), Budne (1959a, 1959b, 1959c), Anscombe (1959),

and Youden, et.al. (1959). It has been suggested that the RB concept

has utility in the design of supersaturated (i.e., N < K) screening

experiments in which the researcher attempts to identify, in a severely

limited number of test runs, the most important factors out of a large

number of possible contributing factors.

A major concern with RB experimental design is that there are no

specific analysis techniques for these designs. Satterthwaite (1959; p. 126)

remarks that practically any technique used to analyze data without RB

properties can be applied in a RB design to analyze any (suitably small)

subset of factors, ignoring all other factors. The simplest approach,

then, would be to consider each factor separately and apply some standard

parametric or nonparametric test of significance. Anscombe (1959) has

suggested that one might make a randomization test, following Welch (1938)

or Tukey (1959), or a normal-theory F-test.

In a recent paper, Mauro and Smith (1982) considered the use of a

standard F-test applied separately to each factor as the method of analysis

for RB designs. To approximate power, the usual normal-theory assumptions

were made. It was found that power calculated under normal theory agreed very closely with corresponding Monte Carlo estimates, even for relatively small values of N. Type-I error probabilities were also found to be in very close agreement with nominal α levels. The overall extent of this agreement was slightly unexpected because it was known that the model error terms did not satisfy the normal-theory assumptions of joint normality and independence. In particular, Scheffé (1959; pp. 331-369) has indicated that correlation in the observations can have a peculiar and often serious effect on inferences about means. It is curious, therefore, why these violations had such little effect in the RB model.

The purpose of this short note is to give a simple explanation for the somewhat surprising results obtained by Mauro and Smith (1982) and to give an improved approximation with which to calculate power probabilities. In our approach, we observe that the RB model (when analyzed with individual F-tests) has exchangeably distributed error terms and we make use of results derived by Arnold (1979, 1981) for the exchangeable linear model (ELM). Our discussion provides an interesting application of the ELM and also serves to explore the theory on which RB experimentation is based.

2. THE EXCHANGEABLE LINEAR MODEL

The random variables e_1, e_2, ..., e_r are said to be <u>exchangeably</u> <u>distributed</u> if the joint distribution of $e_{\pi 1}$, $e_{\pi 2}$, ..., $e_{\pi r}$ is the same as the joint distribution of e_1, e_2, ..., e_r for all permutations π of $(1, 2, ..., r)$.

In the ordinary linear model (OLM) we assume that the error terms are i.i.d. normal random variables. In the ELM we assume exchangeably normally distributed errors. Following Arnold (1981; pp. 232-238), the ELM is equivalently the model in which we observe $\underline{Y} \sim N_r(\underline{\mu}, \sigma^2 \underline{A}(\rho))$, where $\underline{\mu}$ is an rxl mean vector and $\underline{A}(\rho)$ has the following form

$$\underline{A}(\rho) = \begin{bmatrix} 1 & \rho & \cdots & \rho \\ \rho & 1 & \cdots & \rho \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \cdot & & \cdot & \cdot \\ \rho & \rho & \cdots & 1 \end{bmatrix} \quad .$$

A key result derived by Arnold (1981) is that in an ELM one-way analysis of variance, equality of level means can be validly tested with the usual F-tests used in the OLM (i.e., letting $\rho = 0$ in the ELM). We may also note that the ELM is simply a repeated measures model with only one individual.

3. THE RB MODEL

In the analysis of RB data, let us assume the first-order linear model

$$y_i = \beta_0 + \sum_{j=1}^{K} \beta_j x_{ij} + \epsilon_i$$

where y_i is the ith observation ($i = 1, 2, \ldots, N$), x_{ij} is the level (± 1) of the $j^{\underline{th}}$ factor for the $i^{\underline{th}}$ observation, β_j is the (linear) effect of the $j^{\underline{th}}$ factor, and the ϵ_i are i.i.d. $N(0, \sigma_\epsilon^2)$ random disturbances, σ_ϵ^2 unknown. Recall that in an RB experiment, the $j^{\underline{th}}$ design column $\underline{x}_j = (x_{1j}, x_{2j}, \ldots, x_{Nj})'$ is an Nx1 vector consisting of a random arrangement of N/2 +1's and N/2 -1's. By construction, the K column vectors of the design matrix $\underline{X} = (\underline{x}_1, \underline{x}_2, \ldots, \underline{x}_K)$ are independent. Further, we assume that $\underline{X}$ and $\underline{\epsilon} = (\epsilon_1, \epsilon_2, \ldots, \epsilon_N)'$ are independent.

Suppose we wish to test the hypothesis $H_0: \beta_j = 0$ versus $H_1: \beta_j \neq 0$ with a simple F-test (or, equivalently, a two-sample t-test) applied to the observations at the high (+1) and low (-1) levels of the $j^{\underline{th}}$ factor. To simplify notation, suppose we use as prototype $j = 1$. Further, without loss of generality, assume the observations are indexed so that $\{y_i; i \leq N/2\}$ have $x_{i1} = +1$ and $\{y_i; i > N/2\}$ have $x_{i1} = -1$. Let $\underline{Y} = (y_1, y_2, \ldots, y_N)'$.

Thus, for $i \leq N/2$ we have $y_i = \beta_0 + \beta_1 + e_i$, and for $i > N/2$ we have $y_i = \beta_0 - \beta_1 + e_i$ where $e_i = \sum_{j=2}^{K} \beta_j x_{ij} + \epsilon_i$. It is easy to show that $\underline{Y}$ has mean vector $\underline{\mu}$ given by

$$\underline{\mu} = (\beta_0 + \beta_1, \ldots, \beta_0 + \beta_1, \beta_0 - \beta_1, \ldots, \beta_0 - \beta_1)'$$

and variance-covariance matrix $\underline{\Sigma} = (\gamma^2 + \sigma_\epsilon^2)\underline{A}(\rho)$ where $\gamma^2 = \sum_{j=2}^{K} \beta_j^2$ and

$\rho = -\gamma^2/(N-1)(\gamma^2 + \sigma_\epsilon^2)$. We see, therefore, that this model has the same covariance structure as the ELM. Moreover, this correspondence is independent of the sample size. The only difference between these two models is that the errors (e_i) in the RB model are not precisely joint normal.

We suspect, however, that this violation has little effect on the F-test for two reasons: (1) Arnold (1980) has demonstrated asymptotic validity against nonnormality for tests of this type for the repeated measures model, of which the ELM is a special case. (2) Nonnormality generally has a small effect on tests about means in the presence of equal groups sampling, zero skewness, and zero kurtosis. In the RB model, each e_i has zero skewness and kurtosis given by $-2 \sum_{j=2}^{K} \beta_j^4/(\gamma^2 + \sigma_\epsilon^2)^2$, which is clearly dominated by the term in the denominator.

As noted earlier, to determine power probabilities Mauro and Smith (1982) assumed $\rho = 0$ (i.e., independent errors). Under this assumption, the individual F-test has an F-distribution with one numerator degree-of-freedom, N-2 denominator degrees-of-freedom, and noncentrality parameter $\delta = N\beta_1^2/\sigma^2$ where $\sigma^2 = \gamma^2 + \sigma_\epsilon^2$. As Arnold (1981) points out, the correct noncentrality parameter under the ELM is $\delta = N\beta_1^2/[\sigma^2(1-\rho)]$. Mauro and Smith (1982) obtained reasonably good power approximations because $\rho = -\gamma^2/(N-1)(\gamma^2 + \sigma_\epsilon^2)$ is generally small. It is clear, therefore, that improved power probabilities of the separate F-test in the RB model can be obtained by using the factor $(1 - \rho)$ in the denominator of the noncentrality parameter.

4. REFERENCES

Anscombe, F. J. (1959), "Quick Analysis Methods for Random Balance Screening Experiments," Technometrics, 1, 195-209.

Arnold, S. F. (1979), "Linear Models with Exchangeably Distributed Errors," Journal of the American Statistical Association, 74, 194-199.

Arnold, S. F. (1980), "The Asymptotic Validity of Invariant Procedures for the Repeated Measures Model and Multivariate Linear Model," To Appear in Journal of Multivariate Statistics.

Arnold, S. F. (1981), The Theory of Linear Models and Multivariate Analysis, John Wiley & Sons, Inc., New York.

Budne, T. A. (1959a), "The Application of Random Balance Designs," Technometrics, 1, 139-155.

Budne, T. A. (1959b), "Random Balance: Part I -- The Missing Statistical Link in Fact Finding Techniques," Industrial Quality Control, 15, 5-10.

Budne, T. A. (1959c), "Random Balance: Part II -- Techniques of Analysis," Industrial Quality Control, 15, 11-16.

Mauro, C. A. and Smith, D. E. (1982), "Factor Screening in Simulation: Evaluation of Two Strategies Based on Random Balance Sampling," Desmatics, Inc. Technical Report No. 113-7. (To appear in Management Science, February 1984.)

Satterthwaite, F. W. (1959), "Random Balance Experimentation," Technometrics, 1, 111-137.

Scheffé, H. (1959), The Analysis of Variance, John Wiley & Sons, Inc., New York.

Tukey, J. W. (1959), "A Quick Compact Two-Sample Test to Duckworth's Specifications," Technometrics, 1, 31-48.

Welch, B. L. (1938), "On Tests For Homogeneity," Biometrika, 30, 149-158.

Youden, W. J., Kempthorne, O., Tukey, J. W., Box, G. E. P., and Hunter, J. S. (1959), "Discussion of the Papers of Messrs. Satterthwaite and Budne," Technometrics, 1, 157-193.

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In this report the use of separate F-tests as a method of analysis for
Random Balance (RB) designs is discussed. The validity of this test
procedure is shown by relating the RB model to the exchangeable linear
model. For this latter model, the usual F-tests are known to be valid
for certain hypotheses.

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